

Definitions

Term	Notation	Example(s)	We say in English ...
sequence	x_1, \dots, x_n		A sequence x_1 to x_n
summation	$\sum_{i=1}^n x_i$ or $\sum_{i=1}^n x_i$		The sum of the terms of the sequence x_1 to x_n
all reals	\mathbb{R}		The (set of all) real numbers (numbers on the number line)
all integers	\mathbb{Z}		The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	\mathbb{Z}^+		The (set of all) strictly positive integers
all natural numbers	\mathbb{N}		The (set of all) natural numbers. Note: we use the convention that 0 is a natural number.
piecewise rule definition	$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$		Define f of x to be x when x is nonnegative and to be $-x$ when x is negative
function application	$f(7)$ $f(z)$ $f(g(z))$		f of 7 or f applied to 7 or the image of 7 under f f of z or f applied to z or the image of z under f f of g of z or f applied to the result of g applied to z
absolute value	$ -3 $		The absolute value of -3
square root	$\sqrt{9}$		The non-negative square root of 9

Data types

Term	Examples:
<hr/> set unordered collection of elements <i>repetition doesn't matter</i> <i>Equal sets agree on membership of all elements</i> <hr/>	(add additional examples from class) $7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$ <hr/>
n-tuple ordered sequence of elements with n “slots” ($n > 0$) <i>repetition matters, fixed length</i> <i>Equal n-tuples have corresponding components equal</i> <hr/>	
string ordered finite sequence of elements each from specified set <i>repetition matters, arbitrary finite length</i> <i>Equal strings have same length and corresponding characters equal</i>	

Special cases:

When $n = 2$, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted λ .

A set with no elements is called the **empty set** and is denoted $\{\}$ or \emptyset .

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with $\{$ then list elements of the set separated by commas and close with $\}$.

To define a set using **set builder definition**, either form “The set of all x from the universe U such that x is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe U ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol \in as “is an element of” to indicate membership in a set.

Example sets: For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\mathbf{A}, \mathbf{C}, \mathbf{U}, \mathbf{G}\}$$

$$\{\mathbf{AUG}, \mathbf{UAG}, \mathbf{UGA}, \mathbf{UAA}\}$$

Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

New! Cartesian product of sets and set-wise concatenation of sets of strings

Definition: Let X and Y be sets. The **Cartesian product** of X and Y , denoted $X \times Y$, is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

Definition: Let X and Y be sets of strings over the same alphabet. The **set-wise concatenation** of X and Y , denoted $X \circ Y$, is the set of all results of string concatenation xy where $x \in X$ and $y \in Y$

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

Pro-tip: the meaning of writing one element next to another like xy depends on the data-types of x and y . When x and y are strings, the convention is that xy is the result of string concatenation. When x and y are numbers, the convention is that xy is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Fill in the missing entries in the table:

Set	Example elements in this set:
B	A C G U
	(A, C) (U, U)
$B \times \{-1, 0, 1\}$	
$\{-1, 0, 1\} \times B$	(0, 0, 0)
$\{A, C, G, U\} \circ \{A, C, G, U\}$	GGGG

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